

a cloud forms in ordinary impure air only a small proportion of the dust centers become active centers of condensation. He has counted on the Rigi Kulm as many as 4,000 dust particles per cubic centimeter in clouds and 7,700 in dense clouds. While in fog there are as many as 50,000. While it is not probable that the ions could cause the formation of cloud, they might give rise to rain. When the air is in a certain unstable condition any ion more active than others will grow rapidly and falling through highly saturated air will relieve the tension along its path, and we may thus have an active cause in the formation of a raindrop.

From all that precedes, it is evident that the processes at work in the formation of a raindrop are exceedingly intricate, but with a rapidly increasing knowledge of physical relationships it does not seem hopeless to undertake elaborate experiments to determine the active agencies in what may be called the field of collapse. At Mount Tamalpais, as we have tried to show, fog conditions are pronounced, saturated and super-saturated strata lie in close juxtaposition and seemingly are within reach of experimentation.

PRESSURE OF SATURATED AQUEOUS VAPOR AT TEMPERATURES BELOW FREEZING.

By Prof. MAX THIESEN, dated Friedrichshagen, January 12, 1899, from the Ann. d. Phys. u. Chem., March, 1899, vol. 67, pp. 690-695.

The following computations were made in order to investigate how far much more accurate determinations of the pressure of aqueous vapor than at present exist would be of interest at low temperatures. At first the temperature itself was determined, for which the difference in the pressures over water and over ice becomes a maximum, then the absolute pressures themselves for both cases were computed. Some of the relations that resulted in this work will not be without interest to others.

We first establish the equation of condition for the temperature when the maximum difference occurs; that such a maximum in general must occur follows from the fact that the difference between the two conditions over water and over ice is inappreciable both in the neighborhood of 0° C. and also at very low temperatures.

Let v_1 and v_2 be the volumes of the vapor and the fluid at the absolute temperature T ; p the pressure of the saturated vapor in contact with the fluid; ρ the latent heat of evaporation of water; and let the corresponding quantities for ice be indicated by indices; then, according to Clapeyron and Clausius,

$$(1) \quad (v_1 - v_2) \frac{dp}{dT} = \frac{\rho}{T};$$

$$(2) \quad (v'_1 - v'_2) \frac{dp'}{dT} = \frac{\rho'}{T}.$$

When the difference between the two vapor pressures is a maximum, the change or differential of $p - p'$ with respect to T becomes 0; consequently, at this point we have

$$(3) \quad \frac{v'_1 - v'_2}{v_1 - v_2} = \frac{\rho'}{\rho}.$$

An approximation that will be demonstrated hereafter is now introduced into the preceding rigorous formula by the assumption that, corresponding to the Mariott and Gay-Lussac law, we may assume—

$$(4) \quad p(v_1 - v_2) = p'(v'_1 - v'_2) = R T.$$

The equations (1), (2), (3) now become—

$$(5) \quad \frac{d \log p}{dT} = \frac{\rho}{R T^2}$$

$$(6) \quad \frac{d \log p'}{dT} = \frac{\rho'}{R T^2}$$

$$(7) \quad \frac{p}{p'} = \frac{\rho'}{\rho}$$

If, now, we indicate by T_0 the temperature, which lies only a little above 0° C., for which $p' = p$, then by the subtraction of (6) from (5), followed by integration, we have—

$$\log \frac{p}{p'} = - \int_{T_0}^T \frac{\rho' - \rho}{R T^2} dT$$

and by connecting this with equation (7) we have, finally:

$$(8) \quad \log \frac{p'}{\rho} = - \int_{T_0}^T \frac{\rho' - \rho}{R T^2} dT.$$

From this equation the location of the maximum can be determined with sufficient accuracy; to this end the individual quantities entering into the equation must be studied more closely.

The quantity $(\rho' - \rho)$ is the latent heat of liquefaction of ice; it may be designated by σ . The variation of this quantity with temperature and under constant pressure (as we may here assume without appreciable error) is given by the expression—

$$d\sigma = (C - C') dT$$

in case C and C' are the specific heats of water and ice under constant and inappreciable small pressure.

The numerical values of σ , C and C' in the neighborhood of 0° C. are given in calories as follows:

$$(9) \quad \sigma_0 = 79.9; C = 1; C' = 0.474;$$

hence the heat of liquefaction increases for each degree by 0.526 calories, or 0.0066 of its own value, or almost at the same rate as T^2 ; with a little greater accuracy we may write

$$(10) \quad \frac{\sigma}{T^2} = \frac{\sigma_0}{T_0^2} (1 - 0.007 t)$$

where we, for the sake of brevity, have put $t = T - T_0$. The right-hand side of equation (8) has, therefore, the value:

$$- \frac{\sigma_0}{R T_0^2} (1 - 0.0004 t) t.$$

In order to compute the quantity $\sigma_0/R T_0^2$, which occurs herein, we propose the two following methods:

(A.) We convert the above value of σ_0 into mechanical units by multiplying it by 41.34, where the atmospheric pressure is considered as the unit of pressure and compute the value of R from the corresponding value for carbonic acid gas, as it results from Regnault's observations¹, after reduction with the latest values of the atomic weights. We thus obtain:

$$(11) \quad \frac{\sigma_0}{R T_0^2} = \frac{79.9 \times 41.34 \times 0.2200}{273^2} = 0.00975.$$

(B.) By means of equation (5) we rewrite the expression just computed in the following form:

$$\frac{\sigma_0}{R T_0^2} = \frac{\sigma}{\rho} \times \left[\frac{d \log p}{dT} \text{ at } 0^\circ \text{ C.} \right]$$

We compute the value of ρ for the temperature 0° C., from the expression²

$$(12) \quad \rho = r (\tau - T)^3$$

where³ $\log. r = 1.9214$, $\tau - T_0 = 365$, whence $\rho_0 = 596.3$ (calories.)

For the computation of the second factor I use the following equation:

¹ M. Thiesen. Wiedemann's Annalen. 1885. Vol. XXIV, p. 483.

² M. Thiesen Sitzungsab. d. Phys. Gesell. zu Berlin. 1897. Vol. XVI, p. 80.

³ I uniformly designate the natural logarithms by \log and the ordinary Briggian logarithms by $\text{Log}.$

$$(13) \quad T \log p = 5.409 (t-100) - 0.508 \times 10^{-8} \\ \times \left\{ (365-t)^4 - 265^4 \right\}$$

in which t is measured from 0°C .

This equation, which only contains two constants to be determined from the observations of vapor tension, represents the best observations between 0°C . and 180°C . quite well; from it there follows:

$$\frac{d \log p}{d T} = 0.07268 \text{ at } 0^\circ \text{C}.$$

Hence we obtain by our second method the value:

$$\frac{\sigma_0}{R T_0} = \frac{79.9}{596.3} \times 0.07268 = 0.00974$$

The perfect agreement of this second with the preceding value demonstrates the applicability of the Avogadro law at 0°C . and makes quite probable the assumption of the applicability of Mariott's law, which was assumed in deducing equation (5).

The still remaining value of ρ' in equation (8) is the sum of ρ and σ for 0°C . and is equal to 676.2. Its variation with temperature at 0°C . is equal to $-0.545 + 0.526 = -0.019$ for each degree centigrade and can be neglected in the present problem. This is materially smaller than the change of the density of ice with temperature; a still greater constancy [in the change of ρ' with temperature] results from the agreement of the values of the specific heat of ice and vapor as observed by Regnault.

If now we sum up all our results we find that the equation of condition (8) takes such a form that t is a quantity that depends on quantities that vary slowly with t . Therefore we easily find for t an approximate value, and after computing for this approximate value the slowly varying quantities, we find the exact value of t . The execution of this computation gives the following:

$$(14) \quad -t = \frac{\log \frac{676.2}{602.7}}{0.00975 \times 1.005} = 11.7^\circ.$$

Consequently the extrapolation of the quantities that enter into the computation extends to a few degrees only, and the computed value can be considered as quite certain. Fischer, in his observations, had already come so near to this temperature⁴ that he certainly would, at lower temperatures, have observed no appreciably greater difference between the tension of saturated vapor over ice and over water.

We now proceed to the attempt to compute the tensions themselves and first the tension of vapor over ice. From equation (6) by integration, considering the above found approximate constancy of ρ' as being perfect, there results

$$(15) \quad \log \frac{p'}{p_0} = \frac{\rho'}{R T_0} \cdot t$$

and if we substitute the above found value and pass to common logarithms, we obtain

$$(16) \quad \text{Log } \frac{p'}{p_0} = 9.78 \frac{t}{T}.$$

The value of $\text{Log } p_0$ resulting from equation (13) is -2.2198 , or $+0.6610$ in case we adopt a millimeter of mercury as the unit of pressure. With this value, the values of p' , given in the table below have been computed by the use of equation (16).

In the computation of the vapor tension over water the value of ρ given by equation (12) is to be substituted in equation (5) in order to be consistent with the assumptions of this present article. The result of the integration of the equation thus obtained can be expressed in definite form, but

a development in series is preferable. For high temperatures the result will then certainly be incorrect, since the law of Mariott then no longer obtains; but for very low temperatures this is of no importance, since at these temperatures it is no longer possible to keep the water in its fluid condition; therefore we give the development that form which seems most appropriate for temperatures in the neighborhood of 0°C , viz:

$$(17) \quad \log \frac{p}{p_0} = \frac{\rho_0}{R T_0} \cdot \frac{t}{T} \left(1 - \frac{t}{6(\tau - T_0)} + \frac{3\tau - 5T_0}{18 T_0 (\tau - T_0)^2} t^2 - \dots \right)$$

or after the substitution of the numerical values:

$$(18) \quad \text{Log } \frac{p}{p_0} = \frac{t}{T} (8.628 - 0.00394t + 0.000002t^2 - \dots).$$

The values of p given below have been computed by this formula.

As above stated the empirical formula (13) represents the observations of vapor tension at the higher temperatures very well. But it would appear *a priori* improper to apply it to very low temperatures, since this formula does not harmonize well with the assumption of the applicability of the Mariott law. It is therefore necessary to investigate to what extent the continuity is practically preserved in the computation of p by the two formulæ (13) and (18). We therefore rewrite equation (13) in the same form as equation (18) and it becomes:

$$(19) \quad \text{Log } \frac{p}{p_0} = \frac{t}{T} (8.617 - 0.00406t + 0.000007t^2 - \dots).$$

The difference between the values of p computed by formulæ (19) and (18) attains a maximum of about 0.0016^{mm} of mercury in the neighborhood of $t = -13.6^\circ \text{C}$. Therefore, in consideration of the accuracy of our present knowledge of the observed vapor tensions, we may consider formulæ (18) and (19) as identical for temperatures below 0°C .

Of all the assumptions that underlie the computation of the values of p and p' in the following table, that of the vapor tension at 0°C . is the most uncertain. However, even a perfectly accurate new determination for the lower temperatures would scarcely give anything more than an improvement on this value, but it would of course be more desirable to determine it directly. The values of p' for very low temperatures are of interest in questions of cosmic physics.

Pressures of saturated aqueous vapor expressed in millimeters of mercury.

t .	Over ice. p' .	Over water. p .
$^\circ \text{C}$.	Mm.	Mm.
0	4.51	4.581
-5	3.010	3.163
-10	1.946	2.145
-11.7	1.672	1.878
-15	1.237	1.432
-20	0.772	0.989
-25	0.473	0.604
-30	0.294
-35	0.167
-40	0.096
-45	0.054
-50	0.029
-55	0.016
-60	0.008
-65	0.004
-70	0.002
-75	0.0009
-80	0.0004

AURORAL OBSERVATIONS ON THE SECOND WELLMANN EXPEDITION MADE IN THE NEIGHBORHOOD OF FRANZ JOSEF LAND.

By EVELYN B. BALDWIN, Observer, Weather Bureau.

A very complete record of auroral phenomena was kept by

⁴ W. Fischer, Wiedemann's Annalen, 1886, Vol. XXVIII, p. 400.